On the rate of claims inflation for XL contracts with European stabilisation clauses

St. Reimann [1]
Swiss Re, Risk and Analytics Department
Mythenquai 50/60 8022 Zuerich, CH

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[1]email: stefan_reimann@swissre.com

Abstract

Future payments particularly of long-tailed P&C business, are subject to inflation. While reserves are built based on inflation expectations, unexpected future inflation challenges initially built reserves. The impact of unexpected inflation on unexpected payment depends on the contract and the line of business. In this sense we say that each contract has a particular sensitivity with respect to unexpected inflation. Sensitivtity defined as the elasticity of claims payment with respect to inflation can be regarded as a rate of claims inflation. This note aims to develop a unifying picture about contracts, their risk and risk mitigation by contract structure for an excess-of-loss contract with finite retention and cover subject to an European stabilisation clause.

Keywords: risk mitigation, Excess-of-loss contract, stabilisation clause

1 Introduction

An insurer's exposure to inflation risk is distinctive in that an insurance contract translates into an obligation to compensate for the *value* of a claim by paying adequate goods and services at some point in future. Since economic inflation changes the *value* of a liability or an asset, economic inflation induces claims inflation. Obviously inflation has a cumulative effect and thus affects business stronger the more longer-tailed this business is. In this note we do not consider this cumulative effect unfolding over time since we focus on the exposure.

While *expected* inflation is captured in the best estimates of ultimate payments in the pricing of contracts, *unexpected* inflation, i.e. deviations from expected inflation, is a risk factor challenging the adequacy of expected payments and hence finally reserves. In real business, the exposure is an aggregate of a variety of contracts in different Lines of Business. Payment rules and legal conditions of these contracts are highly diverse while portfolios are large. This makes a strict bottom-up derivation of properties of the exposure map difficult in general. Consequently we are left to modeling characteristic features of these contracts to get a reasonable representation of the exposure map.

Non-proportional contracts typically are excess-of-loss contracts with finite retention, a cover to limit maximal payment and index clauses. When modeling P&C exposure we have to consider these features. We start with considering single Excess-of-Loss contracts and derive explicit formulas for excess-of-loss contracts with finite retention for a wide class of loss distributions, Observation 1. The sensitivity of a portfolio is shown to be the weighted average of the sensitivity' of the single contracts within this portfolio, Observation 2. Contract structures decreasing sensitivity are finite covers and stabilisation clauses. Their interplay is described in Observation 3. This is our central result to discuss risk mitigation by contract structure.

2 Sensitivity and claims inflation

While reserves are built according to our best estimates of future payments, reserves include expectations about inflation. If the actual inflation deviates from our expectation, then reserves have to be adjusted. What is the corresponding extent of this adjustment? Typically the question is phrased as follow: *Given that at the end of the year inflation deviates from what is expected by n base points, how much does actual payment then deviate from the expected?'*

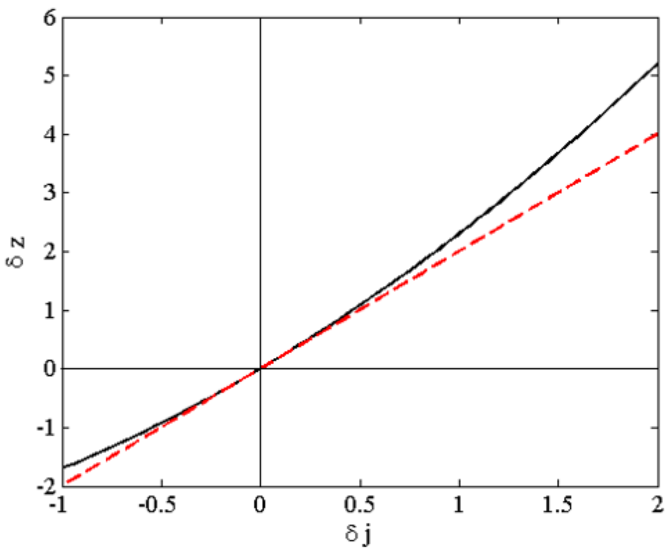
The framework for the following is a 1-year view. At present time 0 we have some expectation $\hat{j}$ about future inflation rate at time 1. If the finally realised inflation rate at time 1 is j, then the relative deviation of the realised inflation rate from the initially expected one is

$$\delta j := \frac{j - \hat{j}}{\hat{j}}. \tag{1}$$

We may call δr the rate of unexpected inflation. Analogously, at the beginning of the year the expected payment is $\hat{z}$, while at the end of the year the realised payment is z. We define the *unexpected payment rate* as

$$\delta z := \frac{z - \hat{z}}{\hat{z}} \tag{2}$$

The question is about the relative change in payment caused by unexpected inflation. This is a question about how changing one variable affects an other. It is therefore natu-



ral to define it as an *elasticity* and to quantify it as the ratio of the relative change in one variable, e.g. unexpected inflation, to the relative change in the other, e.g. unexpected payment.

The *sensitivity* of an exposure is defined as

$$R := \frac{\delta z}{\delta j}\Big|_{\delta z = 0} \tag{3}$$

Note that according to this definition $z = \hat{z}(1+R\delta j)$ so that $R\delta j$ might be regarded as the *rate of claims inflation*. The following ranges of values are commonly used in the insurance business to characterise the loss potential of particular Lines of Business [6].

loss potential	sensitivity
earthquake	0.5 - 0.8 (1)
European Windstorm	1.3
fire	1.5 - 2.5 (2)[1]
motor liability	1.5 - 3.0 (2.5)
general liability	1.0 - 2.0 (1.8)
occupational injury	(2)

3 The sensitivity of a XL contract

We start our discussion by investigating a single Excess-of-Loss contract with finite retention $r > 0$ and infinite cover $\kappa = \infty$ without stabilisation clause. Such contracts are strongly exposed to inflation and serve as an upper bond for inflation sensitivity since finite covers as well as stabilisation clauses would reduce sensitivity.

Observation 1 (sensitivity)**.** *Let losses be distributed according to some distribution φ, whose first moment exists. Then the sensitivity of an Excess-of-Loss contract with retention $r \geq 0$ and infinite cover is*

$$R_r^\infty(\varphi) = 1 + \frac{r}{\mathbb{E}[(X-r|X \geq r)]} \tag{4}$$

Proof. Let claim sizes C be distributed according to some φ, whose first moment exists. We assume that $j = \hat{j} + \delta j$, where $|\delta j| < 1$ is sufficiently small. Further let $X = jC$ and $Y = \hat{j}C$, so that $X = (1+\delta j)Y$. Expected payment is $\mathbb{E}(X-r)^+ = z(\delta j) =: (1+\delta j)\mathbb{E}\left(Y - \frac{r}{1+\delta j}\right)^+$. Expanding the expected payment in $\delta j = 0$ yields $z(\delta j) = z(0) + z'(0)\delta j + \frac{1}{2}z''(0)(\delta j)^2 + \mathscr{O}\left((\delta j)^3\right)$ Note that $z''(0) = \Big(r\varphi(r)\Big)^2 \geq 0$. Respecting $z(0) = \hat{z}$, we obtain $\delta z = \frac{z(\delta j)-\hat{z}}{\hat{z}} = \frac{z'(0)}{z(0)}\delta j$. Thus $R_r^\infty(\varphi) = \frac{z'(0)}{z(0)}$. Using $z(\delta j) = (1+\delta j)\int_r^\infty (y-r)\varphi(y)dy$, we obtain

$$R_r^\infty(\varphi) = \frac{\int_r^\infty y\varphi(y)dy}{\int_r^\infty (y-r)\varphi(y)dy} \tag{5}$$

from which we deduce eq 4. □

In case of a finite cover $\kappa > r$, putting $\mathscr{K}_r^\kappa := (\kappa - r)\int_r^\infty \varphi(y)dy$, eq 5 becomes

$$R_r^\kappa(\varphi) = \frac{\int_r^\kappa y\varphi(y)dy}{\int_r^\kappa (y-r)\varphi(y)dy + \mathscr{K}_r^\kappa}. \tag{6}$$

Obviously, $R_r^\kappa(\varphi) \nearrow R_r^\infty(\varphi)$, i.e. a finite cover reduces sensitivity.

Note that the sensitivity is identical to the logarithmic derivative of expected payment, i.e. $R_r^\infty(\varphi) = \frac{z'(0)}{z(0)}$, i.e. it is the elasticity of the expected excess payment. The sensitivity of a contract indeed depends on the 'balance' between retention on the one hand and the tail behavior on the other: if, for example, the claims distribution were thin-tailed while the retention is large, the contract would hardly be affected by inflation. Here is a list of exmples:

- *The* PARETO DISTRIBUTION $\varphi_\alpha(y) = \alpha\frac{\hat{y}^\alpha}{y^{1+\alpha}}$ *where* $\alpha > 1$ *and* $y \geq \hat{y}$ *is commonly used in reinsurance for modeling claims distribution [3]. In this case the sensitivity is constant*
$$R_r^\infty(\varphi) = \alpha. \tag{7}$$
This result is due to the scale-invariance of this distribution. Since no characteric scale exists, sensitivity has to be independent of any finite retention and cover.

- *The* GENERALISED PARETO DISTRIBUTION *provides a smooth cross-over from the Pareto to the exponential distribution. It is the limiting distribution of the exceedance of a random variable for a large class of underlying distributions [4],[7]. Moreover, the Generalised Pareto can be obtained by the mixture of the exponential with a Gamma distribution. Due to the lack of scale-invariance, sensitivity depends on the retention. Assume that losses are approximately distributed according to* $\varphi(y) = \frac{1}{\sigma}\left(1 + \frac{1}{\alpha}\left(\frac{y}{\sigma}\right)\right)^{-(1+\alpha)}$ *with parameters* $\alpha > 1$, $\sigma > 0$. *Then the sensitivity of the contract is*
$$R_r^\infty(\varphi) = \frac{\alpha\sigma - \mu + \alpha r}{\alpha\sigma - \mu + r} \leq \alpha, \tag{8}$$
while equality holds asymptotically, i.e. $r \to \infty$.

- *Let claim's value Y be* EXPONENTIALLY DISTRIBUTED *with density* $\varphi(y) = \alpha e^{-\alpha y}$, $\alpha > 0$, *so that the average claim's value is* $\mathbb{E}[Y]$ *is* $1/\alpha$. *A simple calculation reveals that* $R_r^\infty(\varphi) = 1 + \alpha r$, *i.e. sensitivity increases linearly with the retention.*

- *The* BENKTANDER-WEIBULL *distribution* $\varphi_{\alpha,b}(y) = \frac{dB_{\alpha,a}(y)}{dy}$ *is used in liability business. For motor liability business* $b = 1/2$ *has proved useful. Its asymptotic* *behavior lies between the exponential and the Pareto*[2]*. Its sensitivity yields*

$$R_r^{\infty}(\varphi) = 1 + (\alpha - 1)\, r^b,\ 0 < b \leq 1, \tag{9}$$

which is concave in r. No upper limit exists.

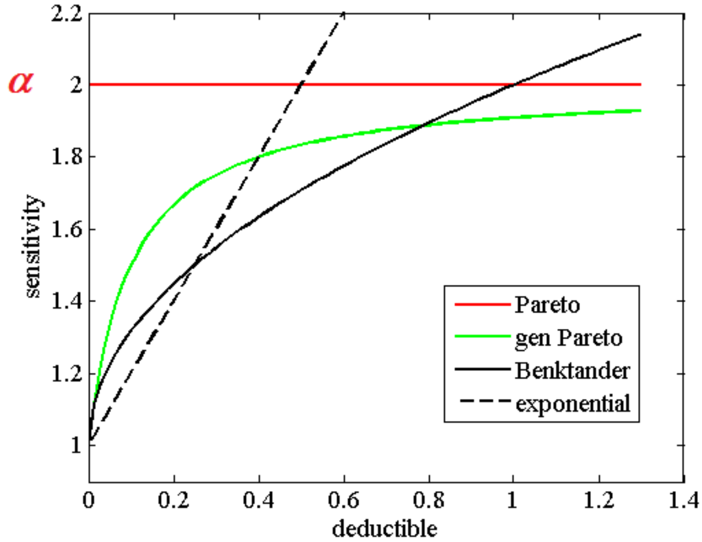


Figure 1: Does the Pareto-α provide an upper bound for sensitivity?

The above examples suggest that α is not an upper bound for loss distributions whose tails decreases faster than polynomial. If the loss distribution decays (asymptotically) as a power-law the respective tail parameter serves as an upper bound.

3.1 The sensitivity of a portfolio

Often cashflows come from a portfolio rather than from a single contract. It turns out that the sensitivity of the portfolio is the weighted avarage of the sensitivity' of the collection of constituting contracts. We consider a portfolio of XL contracts in a particular line of business. In this case the assumption might be reasonable that all contracts are exposed to the same inflation driver. In this case the individual expected payment

[2]The Benktander distribution (-Weibull) of second kind is most conveniently defined via its distribution function $B_{\alpha,b}(y) = 1 - y^{b-1} \exp\left(-\frac{\alpha-1}{b}(y^b - 1)\right)$ for $y > 1$, $\alpha > 1$ and $0 < b \leq 1$. While $\varphi_{\alpha,1} = (\alpha-1)e^{-(\alpha-1)(y-1)}$ is an exponential distribution , $\lim_{b \to 0} \varphi_{\alpha,b}(y) = \frac{\alpha}{y^{1+\alpha}}$ is a Pareto density with tail index α.

of contract i is $z_i = \mathbb{E}(Y_i - r_i)^+ = (1+\delta j)\left(Y_i + \frac{r_i}{1+\delta j}\right)^+ =: z_i(\delta j)$. Then the aggregate expected payment yields $z = \sum_i z_i(\delta j)$. Again, expanding around $\delta j = 0$ and respecting $\hat{z} := \sum_i z_i(0)$ as above, we obtain $z = \hat{z} + \sum_i z_i'(0)\,\delta j + o((\delta j)^2)$ so that finally $\delta z = \sum_i \frac{z_i'(0)}{\hat{z}}\,\delta j$. For notational convenience let $R^{(i)} := R^{\infty}_{r_i}(\varphi_i)$ denote the sensitivity of contract i and $\zeta_i := \frac{z_i(0)}{\sum_j z_j(0)}$ its weight in the aggregated payment of the portfolio.

Observation 2. *The sensitivity of the portfolio is the convex sum of the individual sensitivity' of the contracts involved*

$$R \;=\; \sum_i \zeta_i R^{(i)} \qquad \textit{with weights} \qquad \zeta_i = \frac{z_i(0)}{\sum_j z_j(0)} \tag{10}$$

3.2 Estimating the sensitivity from data

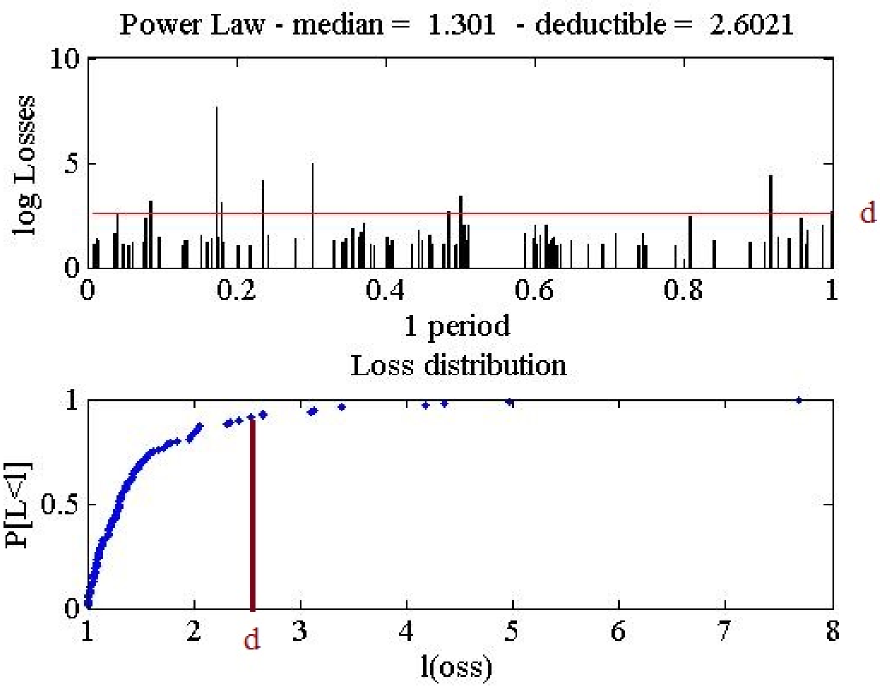


Figure 2: Within one observation period, 100 events happen, whose sizes are randomly drawn from a Pareto distribution; the corresponding cumulative loss distribution is shown in the left lower figure.

Given a contract, its sensitivity may be estimated directly from data. Whether this is possible or not depends on the data available, of course. Based on the theory outlined above, there are (at least) two ways to estimate the sensitivity numerically. For the sake of simplicity we demonstrate both for the case that the loss distribution is Pareto φ_α some tail index $\alpha > 1$. In this case the sensitivity is known to be equal to α, independent of the retention. Thus we can compare the numerical results with the correct theoretical result. Since for the Pareto retention does not matter, we are free to choose $r = 2 \cdot \text{median}(Y)$ in the numerical example. Given the loss distribution φ_α, we draw a random sample of independent claim sizes $Y_i \sim \varphi_\alpha$.

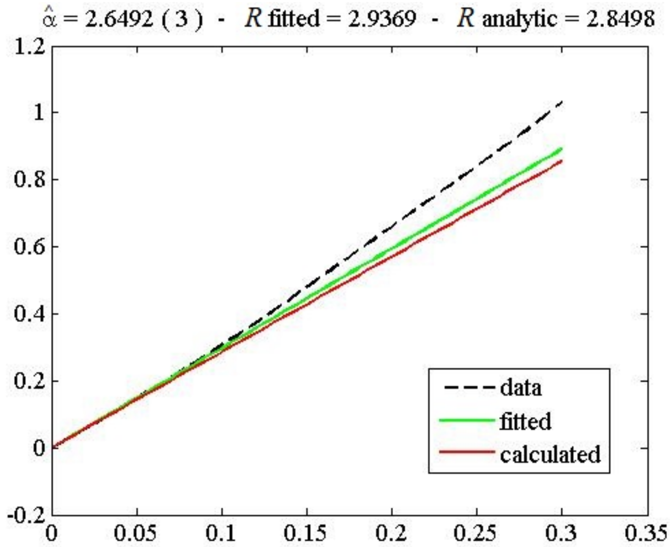


Figure 3: The empirically estimated tail parameter $\hat{\alpha}$, while estimated sensitivity' are S_{fitted} and $S_{analytic}$ in the example above. Recall that only 300 sample points are used.

We now estimate the respective sensitivity along two ways: First, we estimate the sensitivity from the respective sample means according to eqn 4 to get $R_{analytic}$. Second we calculate the sensitivity from scaling the expected payment: For each factor $j \geq 1$ expected payment is

$$z(j) = \mathbb{E}\Big(jY_i - r\Big)^+ = j\,\mathbb{E}\left(Y - \frac{r}{j}\right)^+ \tag{11}$$

For some j_0 and $z(j_0)$, we calculate relative changes with respect to $z(j_0)$, i.e. $\delta z = \frac{z(j)}{z(j_0)} - 1$. Then sensitivity R_{fitted} is found by regressing δz against $\delta j = \frac{j}{j_0} - 1$ close to 0. Results are shown in the Figures below.

4 The sensitivity of an XL contract with stabilisation clause

In the last section, we restricted ourselves to XL contracts without stability clauses. On the other hand, clauses are relevant to losses that are of long-tail nature and are commonly found in other terms and conditions of Motor Liability (MTPL), General Liability (GTPL), and Professional Liability TPL Xl reinsurance contracts of European cedants. Stabilisation Clauses, also called Inflation Clauses or Indexation Clauses, are designed to distribute inflation-related increases in costs of claims, which would fall on the reinsurer only, between the ceding insurer and the reinsurer. For thoughtful insights see [2]. The fundamental assumption is that *all payments in the future carry an inflationary component, which follows the development of an* agreed-on *index K. This component now has to be taken out by readjusting the payments (*).* The value of an index clause is apparent in situations where inflation causes the cost of the claim to reach and exceed the retention amount sooner and more frequently than anticipated.

While European reinsurance contracts began incorporating index clauses heavily in the 1970s, United States-based companies do not appear to have adopted such clauses to protect against inflation but use different methods. There are two basic types of clauses currently used: The *European Index Clause* [EIC] applies the clause index at each date of payment and therefore distributes the effect of inflation in line with the payout pattern due to the claim; Second, the *London Market Indexation Clause* [LMIC] which indexes the total value of the claim at the date of the final settlement. Both have variations, namely the *Franchise Inflation Clause* or the *Serve Inflation Clause* [SIC]. These variations will be analyzed for the EIS.

4.1 Full Index European Index Clause

In the derivation of the payout pattern over time we follow B.J.J. Alting von Geusau [1]. Let A_t be the payment in year t, while C_t and R_t are the amounts of money to be paid by the ceding company and by the reinsurer, respectively. Obviously $A_t = C_t + R_t$. Both parties agree on the retention d, some real index K_t as well as on the physical years which serves as time 0. Therefore K_0 is determined as a reference. For simplicity of calculation the index is normalised so that $K_0 = 1$, while also $A_0 = R_0 = C_0 = 0$. While A_t is supposed to carry inflation due to K_t, this component is taken out by readjusting the payments according to the reference index K. This is done by defining the nominal (readjusted) payment $A_s^{(K)} := \frac{A_s}{K_s}$. This leads to a distribution of inflation between the two parties. Accordingly the reinsurer's share of the claim's value at time t is $\eta_t =$

$\left(1-\frac{d}{\sum_{s=1}^t A_s^{(K)}}\right)^+$. Denoting the cumulative payment during $[0,t]$ by $A(t) := \sum_{s=1}^t A_s$ and defining

$$k_t := \frac{\sum_{s=1}^t A_s}{\sum_{s=1}^t \frac{A_s}{K_s}} = \frac{\sum_{s=1}^t A_s}{\sum_{s=1}^t A_s^{(K)}} \tag{12}$$

we obtain for the reinsurer's share $\eta_t = \left(1-k_t\frac{d}{A(t)}\right)^+$. The reinsurer's payment at time t finally yields

$$R_t = \eta_t A(t) - \eta_{t-1}A(t-1) \tag{13}$$

$$= \left(A(t)-k_t d\right)^+ - \left(A(t-1)-k_{t-1}d\right)^+, \qquad t=1,2,... \tag{14}$$

where $k_0 = 0$ and $A_0 = R_0 = C_0 = 0$. Recall that sensitivity is defined within a One-year

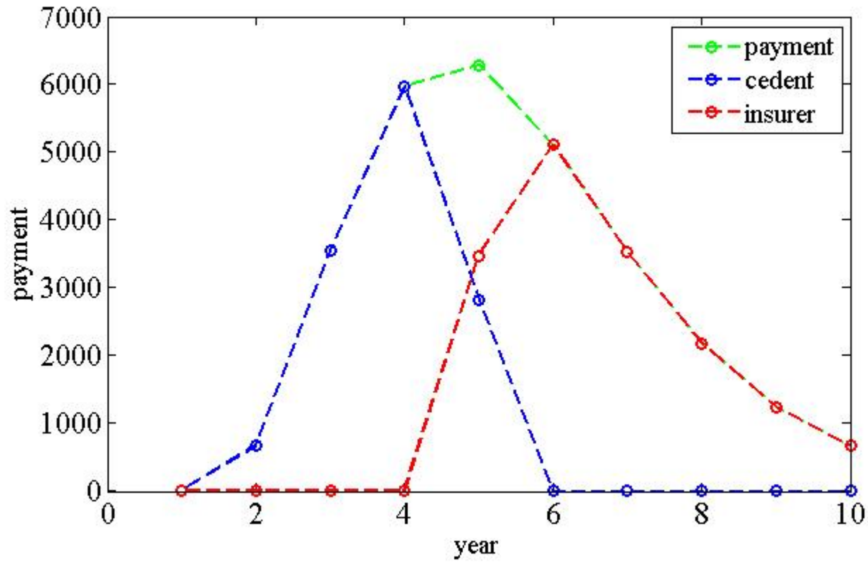


Figure 4: Dynamics of the distribution of payment between cedant (blue) and reinsurer (red) in the case of a Serve Iinflation Clause [20%].

View, i.e. we are interested in payment at time $t=1$, so that

$$R_1 = \left(A_1 - k_1 d\right)^+ \tag{15}$$

For a Full Index European Index Clause $k_1 = K_1$ according to eq 12. Other stabilisation clauses are described in the following.

4.2 Franchise and Serve Inflation Clause

In the last section we described the Standard SC which uses the index K as it is. For this reason this SC is also called a *Full Index SC*. The two subtypes *Franchise SC* (called *Corridor* in von Geusau) and *Severe Inflation Clause* [SIC] differ from the Full Index SC in that not the index K is considered directly but some threshold function of it K^{θ}. In both cases the idea is that only in a high-inflation regime the clause becomes operational.

Again, let the *objective* index be denoted by K and define some threshold θ. Then define K^{θ} as some threshold function of K as displayed in the table below. Note that in low inflation regimes, none of the clauses become operational since for $K_t \leq \theta$, $K_t^{\theta} = 1$.

Type of SC	K_t^{θ}
without	$K_t^{\theta} = 1$
franchise	$K_t^{\theta} = \begin{cases} 1 & \text{if } K_t \leq \theta \\ K_t & \text{otherwise} \end{cases}$
SIC	$K_t^{\theta} = \begin{cases} 1 & \text{if } K_t \leq \theta \\ \frac{K_t}{\theta} & \text{otherwise} \end{cases}$
full	$K_t^{\theta} = K_t$

Table 1: Franchise and SIC are basic non-standard stabilisation clauses. Both react to super-threshold inflation $K_t > \theta$ by adjusting the retention according to some reference index K.

4.3 Impact of a stabilisation clause on sensitivity

Since we are interested only in payments at time 1, we skip the sup script in eq. 15 to obtain $R = (A - kd)$, where $k = 1 + \delta k$ is the clause index at time 1.

Observation 3. *The sensitivity of a XL contract with retention r, cover κ and a stabilisation clause whose clause index k has rate δk is*

$$\mathscr{R}_r^{\kappa}(\delta k) \quad = \quad 1 + \left(R_r^{\kappa}(\varphi) - 1\right)\left(1 - \frac{\delta k}{\delta j}\right). \tag{16}$$

Equation 16 follows from equation 15 by first substituting $k_1 = 1 + \delta k$, which leads to $z = z(\delta j, \delta k) = (1 + \delta j) \int_{\frac{(1+\delta k)d}{1+\delta j}}^{\kappa} dy \varphi(y) \left(y - \frac{(1+\delta k)d}{1+\delta J}\right)$. Second, a first order approximation of $z(\delta d, \delta k)$ in $(0,0)$ and, third, using eq 4 gives the result. We omit straightforward details here. The percentage change of sensitivity due to a stabilisation clause with clause index k is

$$\begin{aligned} \delta \mathscr{R}_r^\kappa(\delta k) &= \frac{\mathscr{R}_r^\kappa(\delta k) - R_r^\kappa(\varphi)}{R_r^\kappa(\varphi)} && (17) \\ &= -\frac{\delta k}{\delta j}\left(\frac{R_r^\kappa(\varphi) - 1}{R_r^\kappa(\varphi)}\right). && (18) \end{aligned}$$

If the clause index moves in the same direction as the inflation index, responsivness is decreased, i.e. the exposure with respect to inflation is decreased. This way a well chosen clause index is said to stabilise a contract. In fact, a stabilisation clause can make a non-proportional contract behave as a proportional one if the rate of the clause index equals the rate of unexpected inflation, i.e. if it perfectly matches the inflation the contract's payment is subject to. The impact of the stabilisation clause depends on the sensitivity of the contract: The higher its sensitivity is the more valuable can a 'good' index clause be for mitigating inflation risk. It follows from the last item that the impact of a mismatch between clause index and inflation is most severe in contracts with high sensitivity. As an example from P & C business, Workers compensation has been a particular case: Seriously injured people lived longer than expected. Since inflation risk is cumulative this significantly increased inflation exposure. Furthermore, if the clause index underestimated inflation in medical cost, the base risk was increasing over time! In summary, policies in workers compensation created huge losses for the insurer.

5 Conclusion

Particularly expected payments of long-tailed business are sensitive to inflation. If inflation at the end of the year deviates from inflation as expected at the beginning of the year, actual reserves may have to be adjusted. Deviations in reserves therefore are proposed to be some function of unexpected inflation. We considered the elasticity of claims payment with respect to inflation and called this term the *sensitivity* of the contract. It furthermore followed from its defintion that the sensitivity equals the rate of correspondiong.

The sensitivity can be calculated explicitly for arbitrary claim's value distributions, whose first moment exists, see Observation 1. While the sensitivity of proportional contracts is equal to one, the sensitivity of non-proportional contracts generally is larger than one. Non-proportional contracts thus enhance inflation risk. sensitivity is larger the more fat-tailed the loss distribution is. Finite covers as well as stabiliation clauses reduce sensitivity. Finally, we consider European Index Clauses and show along which mechanism they tend to stabilize a contract, see Observation 3. Particularly, the impact of an index clause is larger, the larger the sensitivity of the corresponding contract is.

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